



Exact Solutions > Ordinary Differential Equations > Second-Order Linear Ordinary Differential Equations > Second-Order Euler Equation

12. $x^2 y''_{xx} + ax y'_x + by = 0.$

Second-Order Euler equation. Solution:

$$y = \begin{cases} |x|^{\frac{1-a}{2}} (C_1 |x|^\mu + C_2 |x|^{-\mu}) & \text{if } (1-a)^2 > 4b, \\ |x|^{\frac{1-a}{2}} (C_1 + C_2 \ln |x|) & \text{if } (1-a)^2 = 4b, \\ |x|^{\frac{1-a}{2}} [C_1 \sin(\mu \ln |x|) + C_2 \cos(\mu \ln |x|)] & \text{if } (1-a)^2 < 4b, \end{cases}$$

where $\mu = \frac{1}{2} |(1-a)^2 - 4b|^{1/2}$; C_1 and C_2 are arbitrary constants.

References

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

Second-Order Euler Equation

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